Analytic Method

Miloš Kosterec
milos.kosterec@gmail.com

Katedra logiky a metodológie vied
Filozofická fakulta Univerzity Komenského v Bratislave
Gondova 2
814 99 Bratislava

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Abstract
The main aim of this paper is to propose a non-trivial definition of the notion of analytic method. The definition is based on the instructional model of methods. The instructions of methods are divided into three groups: *selective, executive* and *declarative* instructions. All three types of instructions are discussed. The influence of these types of instructions on the analyticity of the methods is discussed. Then the definition of analytic use of the method is provided. The definition of the analytic method follows. Finally I dispute a circularity which is in the presented model if we presuppose a finite agent checking a method for analyticity.

Keywords: analytic method, analytic proposition, closure, instruction, knowledge base

Introduction

The aim of this article[1] is to provide a non-trivial definition of *analytic method*. The definition should be non-trivial i.e. not all methods should be analytic and at least one should be. It should be useful for discerning the methods which are considered analytic. The need for such definition rises from its absence. This notion although commonly used is seldom characterized. Widely used methods such as defining, explication or conceptual analysis are considered analytic. The question is which features make them so.

The main intuition is that an analytic method can be followed without any empirical research. The use of an analytic method enlarges the knowledge base of a researcher without traversing the logical closure of such knowledge base. In other words, we use an analytic method to get, decode, make explicit some information, knowledge which is hidden, encoded or entailed by the information in our knowldege base.

The paper is divided into following chapters. The 1st chapter contains a brief specification of framework this paper presupposes. Three types of instructions are introduced in the 2nd chapter. In the 3rd and th 4th chapters I further specify the selection instructions and the problems of information gain they present for the instructional model of methods. The roots of the problem are described in the 5th chapter and the role of information access is discussed. The role of information access is grasped in th 6th chapter by distinguishing three different types of knowledge bases. The 7th chapter contains the definitions of the analytic instruction and the analytic method. The 8th chapter discusses the role of the declarative instructions. It is followed by a case study about the analyticity of the method of explication in the 9th chapter. The problem connected

1 This paper was supported by Slovak Research and Development Agency on base of ContractNo. APVV-0149-12. I would like to thank my colleagues for all helpfull discussions.

with the finite agent testing method for analyticity is described in the 10th chapter. The paper ends with a brief conclusion.

1 The framework

I presuppose some features of the instructional model of methods presented in Bielik (2014a,b,c,d) and elaborated in Halas (2015). This model depicts a method as a systematic guide for getting some epistemic gain. The use of the method is usually driven by some problem, which can not be resolved within one´s knowledge base. The method is used to change that knowledge base in order to resolve the problem. For example, one does not know whether some formula of propositional logic is a tautology. Then she can use the tableau method for solving this problem.

There is a variety of systems used to model the processes in general (procedural models (Duží (2014)), Petri nets (Murata (1989)) to name a few). Following the instructional model the method is composed of instructions. Instructions are usually expressed by imperatives. I take imperatives to have semantic content sui generis. In general, they denote a relation between input states and output states. I examine the instructions which denote relations among states of knowledge. The features of the method are studied using the compositionality principle in the following form: The relevant features of the method are determined by its instructions and their composition. Therefore the definition of the *analytic method* shall specify some constraints on instructions and/or their composition. In the next chapter I present a typology of instructions which will be later used to specify such constraints.

2 Typology of instructions

According to the instructional model a method is an ordered set of instructions. So far we[2] have studied several methods. Our task was to specify the instructions of those methods. We studied such methods as modelling, defining, explication, idealisation, conceptual analysis,... We developed instructional models for methods of explication, sample picking (see Bielik 2014d), abstraction and idealization (see Halas 2015). All these methods were specified in their idealised form (i.e. without the context of use). They were also described in some case studies. The methods were applied to some particular problems. We stipulated some proto programs for the methods. We then applied those methods on some input arguments. The main result for the purpose of this paper is that we can roughly distinguish the three types of instructions, which occur in methods: *selective*, *executive* and *declarative* instructions.

Selective instructions urge an agent to pick one of the possible ways to continue. For example she can be instructed to pick the number of samples which will be studied later. The selective instructions can also task an agent to choose some arbitrary value which is needed to follow the next steps of the method e.g. to pick some natural number from some range as an input guess which is later improved.

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2 I am a member of the research project Amesh (www.amesh.sk).

The requested selection is seldom completely arbitrary. It usually has some filtering features speficied. For example, compare these two selective instructions:

A) Pick some natural number!
B) Pick some even natural number!

The instruction B is more specific than the instruction A. The selective instructions are very common and provide the main source of indeterminacy in the methods. The method although stated as a set of instructions can nevertheless have its result undetermined.

The second type of instructions are the executive instructions. These instructions provide the actual computation steps of the procedure grasped by the method. The selective instructions urge an agent to produce the input parameters out of her environment. The execution instructions specify how to compute utilizing these parameters. Selection instructions provide the material and executive instructions build using the material. Some examples of executive instructions are:

Find the greatest common divisor of numbers a and b!
Solve the equation E!
Get the median value of the measured results!

Whole methods can be used as executive instructions in other methods. This is very similiar to the programming,where we can reuse a code as a part of some larger code. In logic we can use proofs in other proofs. The executive instructions are the core of the method.

The third type of instructions are the declarative instructions. These instructions guide an agent to declare her results to the public. The term public here is very generic. It can be a classroom full of students or the set of readers of some academic journal. It can also be the agent herself. From a general point of view the size does not matter. The most important feature of these instructions is that the agent assumes the declared results as true in her next work.

3 The information value of selection

Assuming the instructional model of methods there is (among others) a special type of instruction occuring in methods that we focused on. These instructions have at least one common feature. They instruct an agent to pick or select some object out of various possible ones. The agent picks an object or a set of objects. She can be told to pick a material thing, individual or abstract entity. Usually the agent has various possibilities to pick out of. These are provided either by her knowledge or by her actions using that knowledge. In general these instructions are the source of indeterminacy in the method. We can not determine what will be selected (at least not solely on base of the instructions).

The notion of selection from many possibilities is very close to the notion of probability. The theory of information (see Shannon(1948)) assigns informative value to selections. Roughly stated the lower is the

possibility the higher is the information gain if it occurs. The notion of information is very closely related to the notion of knowledge (see Abramsky (2008)). My aim here is to argue that contrary to intuition the selective instructions
do not present any relevant information gain. I do not deny there is some information in the selection. I give reasons why this information is not included into the process of information and knowledge gain provided by the method execution. We are not interested in all of the information about the agent progress. We are rather focused on the change of information state of the knowledge base.

4 The problem of selective instructions

If we include the information provided by the selection instructions into our model, then we have non-determined changes of knowledge base. If we exclude that information, we shall not do that *ad hoc*. This problem arises from the neglecting of the role of information access. In the instructional model of methods we grasped all instructions as knowledge state changers. We did not state the condition for the analyticity. One of the reasons was the problem of the selective instructions. On one hand we know that they do not present relevant information or knowledge gain. On the other hand we model all instructions as possible knowledge changers. We did not deal with the indeterminacy brought in by the selective instructions. Indeterminate knowledge change can not be predicted. Therefore we faced the problem that the use of a selective instruction leads to the crossing of the logical closure of initial explicit knowledge base. Any method containing a selective instruction would therefore not fit the intuitions about the analytic method.

One of the ways how to solve this problem is to leave out the selective instructions when testing the method for analyticity. The main problem of this approach is the need for non-ad hoc reason to do so. The other way is to pick some relevant part of explicit information or knowledge base and test the analyticity of the method only according to the closure on this relevant base. The main problem of this approach is the lack of non-ad hoc reasons to pick the relevant core. We want to check a method for analyticity by checking
the analyticity of its instructions. The first approach would not check selective instructions. The problem was that it was the only reason. The second approach would consider information gain provided by selective instruction i.e. the selection as an irelevant information gain. The problem was that the only reason was that it was the information gain provided by the selective instructions. Either way it is ad hoc. Selection instruction ought to be left out. But why?

5 Information access

The main piece missing in the instructional model of methods is the model of the flow of information among agent, her explicit knowledge base and her operational knowledge base. The difference between explicit an operational base can be described as follows. Imagine yourself sitting at

your notebook. The hard drive of your notebook contains some explicit data. It does not contain all the logical consequences of those data. This does not mean that all your explicit data are accessed all at once after you sign into your account. They are only *accessible, not accessed*. You pick some of the information stored on your hard drive to access it. Then you can make changes. After those you can save them or cancel your work etc. The hard drive is the analogy of the explicit knowledge base. The information you have accessed e.g. some open document, music you play etc. is the analogy of your operational base.

The instructional model of methods neglects this difference between the information which is accessible and the information which is actually accessed and worked with. There is important flow of information between these. The model which would grasp this would not be ad hoc. Notebooks existed long before the discussed model of method.

I will model the information access connected with instructions. The main idea is as follows: Selective instructions provide the access to information from the explicit knowledge. They therefore do not change the informational state of explicit knowledge base. You do not change the information by acessing it. Selective instrutions simply retrieve some of the information already included in the knowledge base and provide it to the operational base. Once again the analogy with your work with notebbok. You simply do not change document by opening it (i.e. by selection). You also do not change the information in database by selecting from it.

6 Executive instructions vs selective instructions

All the instructions provide steps between the informational states of some knowledge base. Itis s common feature of all three types of instructions. But I differentiated the three different knowledge bases. *The explicit knowledge base* of an agent, which could be considered as a base of accessible information. The *operational knowledge base* represents the information already accessed. The *implicit knowledge base* presents the bounds of analyticity of initial explicit knowledge base. The selective instruction provide a step between the state of explicit base to the state of operational base (we select some accessible information). They can also provide a step between the states of operational base (we select some possible way to go). The declarative instruction provide a step from the state of operational base to the state of explicit base. The executive instruction provide a step between
states of operational base.

Now I have to clear the distinction between selective and executive instructions. They seem not to have empty intersection. Selection instruction provide a step between states of operational base simply because we sometimes need to select among possibilities during the computation. I propose to model selective instructions only as steps from states of implicit knowledge base to the states of operational base. We can do this without the loss for the aim specified. In previous chapter, I argued why we need not consider the information provided by selective instruction as relevant. If there is a selective instruction as a step between

two states of operational base, it means that we are at the point of process where previous executive instructions provided us some results. The next steps of the method need the selection among these preliminary results. The selection is not determined. Therefore we must provide it. Now if the possibilities were obtained by analytic instructions, they should be included in the implicit knowledge base of the agent gained by the logical closure of her explicit knowledge base. But then in case of analytic methods we can model selective instructions as steps from the states of implicit knowledge base to the states of operational base. Therefore the sets of selective and executive instructions will be disjoin.

But how can we know whether such transformation is possible. It is possible if all the executive instructions preceding the selective instruction considered are analytic. We can therefore make the transformation without the loss for the purposes of this article. If the method is not analytic, it is because it contains at least one non-analytic executive instruction. Only in that case there is a possibility of failure of the proposed transformation of selective instructions.We therefore need to look only at the executive instructions in order to check the
analyticity of the method and we can differentiate them clearly from the selective instructions.

7 Analytic instruction and analytic method

The differentiation between selective and executive instructions was based on the notion of *the analytic instruction*. This chapter aims to specify this notion. First let´s introduce the notion of *the decriptive result of the executive instruction* (DRE):

The DRE of the executive instruction *I* from the input *a* to the output *b* is *a-I-b*.

I use the DRE to differentiate between the result of the instruction (*b*) and the way the result was obtained (applying instruction *I* on input *a*). Now I can state the condition for the analyticity of executive instruction:

> The executive instruction *I* is analytic iff its DRE is included in the implicit knowledge base.

Until now I presupposed the difference between the explicit and implicit knowledge base. Now its time to consider it carefully.

The need to differentiate between explicit information and its closure is discussed and well established in epistemology (see Dretske (2005)) and informatics (see Vardi (1989)). It is agreed that an agent need not know all the logical consequences of her knowledge (see Jago (2014), chapter 6). The difference between logical entailment and knowledge closure is discussed in the approach of relevant alternatives (see Holliday(2012,2015)). From a technical point of view, the set is closed on operation, if it contains the results of the application of that operation on all its members. For example the set of natural numbers is closed on the operation of addition. There are various closures discussed. The closure of

explicit initial knowledge base presupposed here is the union of *validitiy closure*, *closure on logical entailment* and *closure on semantic analysis*. I furthermore presuppose that explicit initial knowledge base contains all the relevant mathematical and theoretical theories.

The validity closure of the explicit base contains all the sentences which are true whenever all the sentences of the explicit base are true. The logical closure of the explicit base contains all the logical consequences of sentences included in the initial explicit knowledge base. The closure on semantic analysis of the explicit base contains all the relevant semantical parts of the sentences of the explicit base.

There is one specific proposition assignable to each DRE. The proposition is gained in the following way. The semantic analysis of an instruction entails some main semantic operator. For example, the main operator of the following instruction:

I) Add the numbers *a* and *b*!

is the operator of addition (*add*). My task here is not to specify the method for searching for such main operators of instructions. I presuppose that there is such operator for every instruction. For every such operator there is an operation. In my example the operator was *add* and the operation is addition (+). Now we can assign a *descriptive proposition* to every DRE in the following way. We pick DRE and formulate the proposition by replacing the
operator by operation. We get the DRE by describing the result of the execution of the instruction. We get the descriptive proposition by the DRE reformulation. For the example, the description proposition is:

a + b = c.

We can assign descriptive propositions to each execution of instruction in a similiar way.

Now I can finally formulate the definitions. First I define *the analytic use of an instruction*:

Def: The use of instruction *I* is *analytic* iff its descriptive proposition is analytic[3].

The descriptive proposition is assigned to the particular input-output pair provided by the use of the instruction. The instruction usually generates whole relation and not only one such pair. Therefore I must specify more general definition:

Def: The instruction *I* is *analytic* iff all its uses are analytic.

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3 This paper does not specify the notion of *analytic proposition*. It only presupposes it. Some models of analytic propositions are provided e.g by Duží (2010, 2013).

The testing of an instruction for analyticity is by no means an easy task. I discuss some features and possible drawbacks of such testing later. With the two previous definitions in hand I can now specify the conditions of analyticity of the methods taken as an ordered sets of instructions:

Def: The use of method *M* is *analytic* iff all the uses of its executive instructions
are analytic.

The difference between method and its use is that a method is used by its apllication to solve some problem. The instructions are applied and executed. Finally, lets specify the definition of the analytic method which is the main task of this article:

Def: The method *M* is *analytic* iff all its uses are analytic.

How can we test a method for analyticity? We have to analyse its uses. Sometimes it is possible to generalize over them. Then we have to check whether all the executive instructions included in the method lead to some analytic descripitve propositions. We can use DREs to do so.

8 Declarative instructions

The selective instructions provide access to the accessible information. The executive
instructions produce actual computations of the method. The last type of instructions
are the declarative instructions. Should we test the declarative instructions for analyticity?

The general frame of a declarative instruction is:

Declare *A* to be in relation *R* with *B*!

If we make a semantic analysis, we find that the main operator (besides exclamation), is *to declare*. Therefore the main operation provided by a declarative instruction is some declaration or public announcement. The field of public announcement logics is already established (see Baltag (1998), Wang (2013)). The common view is that the public announcement of some proposition changes the situation of an agent in a way that she presupposes the proposition to be true after the announcement. Supposedly there is an update of the agent´s knowledge base after the announcement. I discussed the closures on initial explicit knowledge base of the agent. Now consider the operator of declaration (public announcement). It seems not to be analytic, necessary or determined. Therefore we cannot predict what will be declared.

If we include the declarative instructions among the instructions which should be tested for analyticity in order to check the analyticity of the method the results will be practicaly trivial. All methods containing a declarative instruction will not be analytic. It is similar position as with the selective instrucions. It seems that the declarative instructions belong to

the methods. It also seems that their inclusion leads to the crossing of the limits of the closures of the initial explicit knowledge base.

The remedy could be seen again in consideration of the role of the information access. The difference between selective and executive instruction was that the executive instructions do not operate on explicit knowledge base. They do not access information. They compute with the information already accessed. Now lets look at the declarative instructions. Do they provide any computation with the accessed information? No. They rather provide saving or updating of the explicit knowledge base according to the results gained by the executive instructions. The declarative instructions usually provide a statement which explicitly names or labels some element. Could we leave the PAI out of the testing of analyticity of the methods? We need positive answer based on non ad hoc reasons.

Can we consider the executive instructions of a method to be sufficient in order to provide the requested results? The series of instructions should lead us to the requested result. Let´s look at this from following point of view. First, consider the method as a set of instructions. What is the role of a declarative instruction? It is only to declare the results gained by the previous instructions. The declaration of result is not a result in itself. If the result of some executive instruction is not analytic it will not change after the announcement. The announcement should not change the analyticity of some result. It would be strange to consider some proof as non-analytic just because it was published or announced. The testing of declarative instructions for analyticity therefore seems redundant. If all the executive instructions in the method are analytic the announcement of the result should not change the analyticity of the gained result. We do not need to state the results in order for them to be analytic. On the other hand we do not make the results analytic by announcing them. We therefore should not include the declarative instructions among the instructions which are tested for analyticity. The other way would be self-inflicted wound.

9 Case study

We proposed a model of the method of explication in Bielik (2014d) Here is its simplified version:

1. Declare the content A as explicandum!
2. Select the contents B, C, D which are to be used!
3. If the contents are not clear enough, then clarify them, else construct the
content E out of B, C, D!
4. Test whether the constructed content is theoretically valuable!
5. Declare the content E as explicans of the content A!

The first step in testing of this method for analyticity is to divide the instructions involved into selective, executive and declarative. Clearly, the 1st and the 5th instruction are declarative. These are therefore not tested for analyticity. The 2nd instruction is a selective instruction. It presupposes that we have the contents B, C and D accessible. It only provides us with

the access to these contents. The rest of the instructions, i.e. the 3rd and the 4th are executive instructions. We should study their uses.

Here I rather describe the testing. Lets focus on the 3rd instruction. It is a complex instruction of the form *if ... then ... else ....* We check the instructions in each clause individually. The whole instruction is analytic only if all the instructions in both clauses are analytic. The first clause includes the method of clarification. It is an example of a whole method used in other method. We can state that the analyticity of the method of explication depends on the analyticity of the method of clarification. The second clause contains instruction which leads us to construct a content out of some specified building blocks. The main operator of this instruction is *to construct* and the connected operation is construction. It seems unproblematic that we can construct a content out of specified parts. We can consider this descriptive proposition as analytic. The 3rd instruction is therefore analytic if the used method of clarification is analytic.

The 4th instruction is a testing instruction. It assumes that the conditions of the theoretical value are specified. There is some constructed content on the input. The output is positive or negative answer. Let BCD be the content constructed in previous instruction. The DREs are

BCD - (Test for value) - is valuable.

or

BCD - (Test for value) - is not valuable.

The descriptive propositions are:

BCD is valuable according to the test.

and

BCD is not valuable according to the test.

The analyticity of this instruction obviously depends on the analyticity of the method of testing.

We can see that the method of explication as stated here contains other methods. The analyticity of the method therefore depends on their analyticity. If all the contained methods are analytic then the whole method is analytic.

In this chapter I described how to use the proposed definitions. First we have to select the executive instructions. Then we have to check their uses. If some executive instruction is method in itself, then we must check this method for analyticity.

10 The problem of analytic test

I discussed the types of instructions included in methods and their influence on the analyticity of the methods. I argued that only the executive instructions should be taken into consideration. Provided this, let ´s now focus on one more possible source of problems.

The definition of the analytic instruction is eventually based on the notion of the analytic proposition. We can say that to know how to differ the analytic proposition from non-analytic proposition enables one to

differentiate between analytic and nonanalytic instructions (at least in the presented model). The crucial question here is *how do we test the propositions for analyticity*. The simple of the cuff answer seems to be that we use some testing methods. But this answer raises another question. Should we use analytic methods in order to discern between analytic and non-analytic propositions? The negative answer seems strange. If we use non-analytic tests for testing the analyticity of propositions, we enable our knowledge of their analyticity to be conditioned by empirical factors. On the other hand we could presuppose the use of analytic methods to check the analyticity of the propositions. But then we conditioned the knowledge of the analyticity of those propositions by the analyticity of some method. Isn't it circular?

It is connected with the cardinalities of the set of analytic methods and the set of analytic propositions. It is clear that if we had only one analytic method and only one analytic proposition then our model of analytic methods would be circular. The method would be analytic because the proposition is analytic. We would know that the proposition is analytic because the method is considered analytic. But what if the number of methods is not finite? Then once again after a while we get a circularity. After we used up all the analytic propositions from the finite set we would need another in order to support analyticity of some testing method. So it seems that we should presuppose that not only the set of analytic methods is infinite but also that the set of analytic propositions is infinite as well. This then need not lead to the circularity mentioned. The drawback here is that we are finite entities which can provide only finite tests. We therefore have to consider some method or proposition as analytic without the justification prescribed by the presented model of analyticity of the methods.

The reason for the circularity is that the testing methods for the analyticity of propositions should be analytic. I call this *the problem of analytic test.* Its problem because the notion of analytic method presupposes that we, as finite agents, know at least some analytic propositions beforehand without any test.

11 Conclusion

The main aim of this paper is to provide a definition of the notion of analytic method.
I used the instructional model of methods. I differentiated three different types of instructions. The selective instructions are used to access some accessible information or to provide steps undetermined by the method itself. The executive instructions provide the actual computation steps of the method. The declarative instructions serve for declarations of the results gained by the method.

I conditioned the analyticity of the method by the analyticity of the executive instructions involved. An executive instruction is analytic if its descriptive propositions obtained from its descriptive resulst are analytic. I therefore conditioned the analyticity of methods by the analyticity of propositions. As soon as we consider the role of finite agent in the testing of a method for analyticity we fall into the problem of analytic test.

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